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III. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa., and CHARLES E. MYERS, Canton, O.

Take A as origin. Let (k, l), (y, z) be the coödinates of the center of the sphere when $t=0$ and when $t=t$, respectively, R, the reaction of the plane, P, the friction, a, the radius of the sphere, $\angle DOE=\theta$. Then $AA'=x'$ and $a\theta=x$.

For the sphere the equations of motion are

$\frac{2}{5}ma^2(d^2\theta/dt^2)=aP$..........(1). $m(d^2y/dt^2)=P\text{cos}a-R\text{sin}a$...........(2).

$$m(d^2z/dt^2)=P\text{sin}a+R\text{cos}a-mg\text{..........(3).}$$

For the plane, $M(d^2x'/dt^2)=-P\text{cos}a+R\text{sin}a$........................(4).

By geometry, $y=k+x'-a\theta\text{cos}a$, $z=l-a\theta\text{sin}a$....................(5, 6).

From (5) and (6) we get $d^2y/dt^2=d^2x'/dt^2-a\text{cos}a(d^2\theta/dt^2)$........(7).

$$d^2z/dt^2=-a\text{sin}a(d^2\theta/dt^2)\text{........(8).}$$

(7)$\text{cos}a$+(8)$\text{sin}a$ gives

$\text{cos}a(d^2y/dt^2)+\text{sin}a(d^2z/dt^2)=\text{cos}a(d^2x'/dt^2)-a(d^2\theta/dt^2)$...... ....(9).

(2)$\text{cos}a$+(3)$\text{sin}a$ gives $m\text{cos}a(d^2y/dt^2)+m\text{sin}a(d^2x/dt^2)=P-mg\text{sin}a$......(10).

From (9) and (10), $m\text{cos}a(d^2x'/dt^2)-ma(d^2\theta/dt^2)=P-mg\text{sin}a$.....(11).

(2)+(4) gives $m(d^2y/dt^2)+M(d^2x'/dt^2)=0$................. .....(12).

From (12) and (7) we get $(M+m)(d^2x'/dt^2=ma\text{cos}a(d^2\theta/dt^2)$... ..(13).

The value of P from (1) in (11) gives

$$\tfrac{7}{5}a(d^2\theta/dt^2)-\text{cos}a(d^2x'/dt^2)=g\text{sin}a\text{......(14).}$$

Integrating (13) and (14) and remembering that when $t=0$, $x=0$ and $\theta=0$ we get

$$(M+m)x'=ma\theta\text{cos}a \text{ or } (M+m)x'=mx\text{cos}a,$$

$$\tfrac{7}{5}a\theta-x'\text{cos}a=\tfrac{1}{2}gt\text{sin}a \text{ or } \tfrac{7}{5}x-x'\text{cos}a=\tfrac{1}{2}gt^2\text{sin}a.$$

DIOPHANTINE ANALYSIS.

71. Proposed by A. H. BELL, Hillsboro, Ill.

Find five numbers such that the product of any two plus 1 will equal a square.

I. Solution by CHARLES C. CROSS, Libertytown, Md.

On page 301, Vol. V, of MONTHLY I found four general numbers to be: m, $n^2-1+(m-1)(n-1)^2$, $n(mn+2)$, and $4m(mn^2-mn+2n-1)^2+4(mn^2-mn+2n-1)$.

If we take m and m for the first two numbers, then $m.m+1=\square=y^2$(say) $=(y-s)^2=y^2-2sy+s^2$.

$\therefore$ $y=(s^2+1)/2s$; whence $m=(s^2-1)/2s$.

Let $s=2$, then $m=\frac{3}{4}$. Take $n=2$ and the numbers are $\frac{3}{4}$, $\frac{3}{4}$, $\frac{11}{4}$, 7, and $\frac{315}{4}$.

Take $n=3$ and the numbers are $\frac{3}{4}$, $\frac{3}{4}$, 7, $\frac{51}{4}$, and $\frac{1235}{4}$.

Let $s=3$, then $m=\frac{4}{3}$. Take $n=3$ and the numbers are $\frac{4}{3}$, $\frac{4}{3}$, $\frac{28}{3}$, $\frac{2660}{3}$, etc.

II. Solution by Euler, found in the MSS. of Lagrange by Legendre.

Let the numbers be x, y, z, u, v. To find the first three, such that $xy+1$, $xz+1$, and $yz+1$ shall be squares.

Take any square, l^2, and decompose l^2-1 into any two unequal factors, x and y; then $xy=l^2-1$, and take $z=2l+x+y$.

$\therefore xy+1=l^2$, $xz+1=x(2l+x+y)+1=(l+x)^2$
and $yz+1=y(2l+x+y)+1=(l+y)^2$.

Now take $u=4l(l+x)(l+y)$......(1).

$\therefore xu+1=4lx(l+x)(l+y)+(l^2-xy)^2=(l^2+2lx+xy)^2$,
$yu+1=4ly(l+x)(l+y)+(l^2-xy)=(l^2+2ly+xy)^2$,
$zu+1=4l(l+x)(l+y)(2l+x+y)+(l^2-xy)^2=[3l^2+2l(x+y)+xy]^2$.

To find the fifth number, v, we make an equation of the fourth degree from the roots x, y, z, and u, thus $a^4-pa^3+qa^2-ra+s=0$........(2), from which $v=\dfrac{4r+2p(s+1)}{(s-1)^2}$......(3).

To verify this, $va+1=\square$, $\therefore 4ra+2p(s+1)a+(s-1)^2$ is a square, but in (2) $4ra=4a^4-4pa^3+4qa+4s$.

Now $4a^4-4pa^3+4qa+2p(s+1)a+(s+1)^2=\square$......(4).

When the condition $\frac{1}{4}p^2=q+s+1$......(5) is satisfied the demonstration is completed.

From (2) $p=x+y+z+u$,
$q=xy+(x+y)(z+u)+zu$,
$p^2-4q=(x+y-z-u)^2-4xy-4zu=(2l+u)^2-4(l^2-1)-4zu$,
$4(s+1)=4xyzu+4=4zu(l^2-1)+4$.

$\therefore p^2-4q-4(s+1)=u(u+4l-4l^2z)$.

But in (1) we have $u+4l-4l^2z=0$.

$\therefore p^2-4q-4(s+1)=0$ .....(6) and the condition is proved. This value of $4qa$ in (4) gives the general value

$$va+1=\frac{4a^4-4pa^3+p^2a^2-4a^2(s+1)+2p(s+1)a+(s+1)^2}{(s-1)^2}=\square$$

$$=\frac{(2a^2-pa-s-1)^2}{(s-1)^2}.$$

$$\therefore vx+1=\frac{(2x^2-px-s-1)^2}{(s-1)^2};\quad vy+1=\frac{(2y^2-py-s-1)^2}{(s-1)^2};$$

$$vz+1=\frac{(2z^2-pz-s-1)^2}{(s-1)^2};\quad vu+1=\frac{(2u^2-pu-s-1)^2}{(s-1)^2}.$$

And the five numbers are $xy=l^2-1$, x, y, $2l+x+y$, $4l(l+x)(l+y)$, and $v=\frac{2r+2p(s+1)}{(s-1)^2}$.

$r=xyz+xyu+xzu+yzu$; $s=xyzu$ and $p=x+y+z+u$.

If $l^2=4$ then the five numbers$=1$, 3, 8, 120, and $\frac{777480}{(2879)^2}$.

If $l^2=25$ then the five numbers$=1$, 24, 35, 3480, not carried out.
2, 12, 24, 2380, not carried out.
3, 8, 21, 2080, not carried out.
4, 6, 20, 1980, $\frac{3822388020}{(950399)^2}$.

The Hillsboro Mathematical Club have solved for an integral value by extending the series, without result.
A. H. BELL.

AVERAGE AND PROBABILITY.

65. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

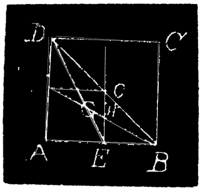
What is the average rate of the sun's motion in declination from the equator to the solstices ?

No solution of this problem has been received.

66. Proposed by REV. W. ALLEN WHITWORTH, A. M.

A rod 9 feet long is to be divided into three parts, of which A is to have the largest, B the next, and C the smallest. If the two fractures are made at random, A's, B's, and C's expectations will be, respectively, 66, 30, and 12 inches. But, if one fracture be made at random and the larger portion of the rod be then divided at random, their expectations will be 64, 31, and 13 inches.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Science and Mathematics, Chester High School, Chester, Pa.



Let $AB=AD=108=a$, $AE=AF=\frac{1}{2}a$.

I. To find the mean value of the least part, we find the values or limits of x, y by restricting the point to the area GOH. The limits for the denominator are given by restricting the point to GOB.

$$\therefore L=\frac{\int_{\frac{1}{3}a}^{\frac{1}{2}a}\int_{\frac{1}{2}(a-x)}^{x} x\,dx\,dy}{\int_{\frac{1}{3}a}^{\frac{1}{2}a}\int_{\frac{1}{2}(a-x)}^{x} dx\,dy+\int_{\frac{1}{2}a}^{a}\int_{\frac{1}{2}(a-x)}^{a-x} dx\,dy}=\frac{\frac{a^3}{108}}{\frac{a^2}{12}}=\tfrac{1}{9}a=12 \text{ inches.}$$

For the mean value of the greatest part the limits are the same for both integrations :